

## CORROSION OF A THIN-WALLED SPHERICAL SHELL UNDER TIME DEPENDENT INTERNAL PRESSURE

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### Abstract.

Elastic thin-walled closed spherical shell is considered exposing to mechanochemical corrosion under constant external pressure and internal pressure decreasing with time. It is important that the equivalent stresses are chosen to be dependent not only on difference of internal and external pressures but on both internal and external pressure values themselves. We are investigating the dependencies of a vessel lifetime on the rate of pressure decrease, under the assumption that corrosion rate depends linearly on stress.

## 1 INTRODUCTION

Thin-walled elastic spherical shells are typical structural component, they are often used as a part of high-pressure vessels. During operation pressure vessels may be under mechanical loads and in aggressive environment. Aggressive environment causes corrosion damage, which, being intensified by stresses in the body, becomes more destructive [1]. Corrosion may cause local defects, for example pittings, cracks, caverns, or may lead to uniform thinning of the structure, i.e. general corrosion. Localized corrosion results in stress concentration around defects, to estimate stresses around local imperfections, finite element method is often used [2], however, there are some analytical assessments [3, 4, 6]. General corrosion accelerated by mechanical stresses is called mechanochemical corrosion. Experiments have shown that for metals, the rate of mechanochemical corrosion is often linearly dependent on stress at the corresponding points of the surface [7, 8].

Mechanochemical corrosion of thin-wall spherical and cylindrical shells under pressure was considered in [9, 10], where the stresses are assumed to follow Laplace's law. According to Laplace's law, stresses are the same on both shell's surfaces and does not take into account the inner and outer pressures, only their difference. However, for pressurized vessels under machenochemical corrosion the use of the law of P.-S. Laplace may under- or overestimate the service life because only pressure difference affects corrosion rate, and

error grows as the inner and outer pressures increase. In [11] an analytical solution for thin pressurized sphere exposed to double-sided corrosion was presented taking into account the effect of hydrostatic pressure and the difference of hoop stresses on the inner and outer surfaces. Thin corroded sphere exposed to nonuniform heating considered in [12]. Several papers devoted to mechanochemical corrosion problems of thick shells [5, 13].

However, solutions mentioned in [8, 10] deal with constant internal and external pressures, whereas another challenging problem is the case of pressure that changes with time. Such problem arises in different areas of engineering, for example, the pressure in oil well declines with time. Pressure decline in a well is often driven by few factors, one of such factors is reservoir pressure depletion.

In the present paper a linearly elastic thin-walled closed spherical shell is considered exposing to mechanochemical corrosion under constant external pressure and internal pressure decreasing with time. The dependencies of the vessel lifetime on the rate of pressure decrease is investigated.

## 2 PROBLEM FORMULATION

Consider a thin-walled spherical shell subjected to pressure and mechanochemical corrosion. It is supposed that sphere's material is linearly elastic. Let the internal pressure  $p_{in}$  linearly decline with time  $t$  while the external pressure  $p_{out}$  remains constant:

$$p_{in} = p_r - at, \quad p_{out} = p_R = \text{const}, \quad (1)$$

in this paper we consider the situations when the parametre  $a$  is positive:  $a > 0$ .

Consider two cases of corrosion damage: internal and external mechanochemical corrosion. Corrosion is understood here as a uniform dissolution along the entire surface such that surface remains spherical with no localized attack. When the shell is subjected to internal corrosion, the inner radius increases with time due to dissolution:  $r = r(t)$ , while the outer radius  $R$  remains constant:  $R = \text{const}$ , the rate of internal corrosion is denoted by  $v_r$ . In case of external corrosion the inner radius of the sphere remains constant:  $r = \text{const}$ , while the outer radius of the sphere is decreasing with time:  $R = R(t)$ , the corrosion rate is denoted by  $v_R$ . Let us denote inner and outer shell's radii at initial time  $t_0 = 0$  by  $r(0) = r_0$  and  $R(0) = R_0$ .

Corrosion rates  $v_r$  and  $v_R$  are supposed to be linearly dependent on the stress at the corresponding surface [7, 8, 14]:

$$v_r = \frac{dr}{dt} = a_r + m_r \sigma(r), \quad (2)$$

$$v_R = -\frac{dR}{dt} = a_R + m_R \sigma(R). \quad (3)$$

Here,  $a_r$ ,  $m_r$ , and  $a_R$ ,  $m_R$  are experimentally determined constants, which are dependent on sphere's material and environment;  $\sigma(r)$  and  $\sigma(R)$  are the maximum (in absolute value)

principal stresses on the corresponding surface of the shell, and

$$\text{sign } m_r = \text{sign } \sigma(r), \quad \text{sign } m_R = \text{sign } \sigma(R).$$

The problem is to find the circumferential stress, shell's thickness and to assess its lifetime.

### 3 PROBLEM SOLUTION

#### 3.1 Equivalent Stress

The stresses defined by the law of P.-S. Laplace do not reflect the effect of the hydrostatic pressure  $p = \min\{p_{in}, p_{out}\}$ , but depend on only the difference  $p_{in} - p_{out}$ . However, for high pressure vessels subjected to mechanochemical corrosion, the use of the law of P.-S. Laplace may lead to a large error [11].

Article [11] presents the refined expressions for stress in a thin-walled pressurized sphere under corrosion which provide results more accurate than the solutions based on Laplace's law, and at the same time, have a simple form (in contrast to the solutions based on based on Lamé's formulas [13]). According to [11] the stress values on the sphere's surfaces determined by the formulas

$$\sigma(r) = \frac{\Delta p r_c}{2h} - \frac{p_{in} + 3p_{out}}{4}, \quad (4)$$

$$\sigma(R) = \frac{\Delta p r_c}{2h} - \frac{3p_{in} + p_{out}}{4}, \quad (5)$$

where  $\Delta p$  is the difference between inner and outer pressures:  $\Delta p = p_{in} - p_{out}$ ,  $h$  is a thickness of the shell,  $r_c$  is the mid-surface radius which is supposed to be constant:  $r_c = (R_0 + r_0)/2 = \text{const}$ . Note that, in general case mid-surface radius of the corroded shell changes with time. However, it is shown in [11] that the change in the mid-surface radius during the corrosion process (even for one-sided corrosion) can be neglected, so the assumption  $r_c = \text{const}$  is justified.

Substituting (1) in Eqs. (4) and (5) yields

$$\sigma(r) = \frac{(p_r - at - p_R)r_c}{2h} - \frac{p_r - at + 3p_R}{4}, \quad (6)$$

$$\sigma(R) = \frac{(p_r - at - p_R)r_c}{2h} - \frac{3p_r - 3at + p_R}{4}. \quad (7)$$

#### 3.2 Internal Corrosion

When the sphere is subjected to internal mechanochemical corrosion, the inner radius increases with time due to corrosion:  $r = r(t)$ , while the outer radius  $R$  remains constant:  $R = R_0 = \text{const}$ . The shell thickness  $h$  at any time is defined as

$$h = h(t) = R_0 - r(t). \quad (8)$$

Substituting Eqs. (6) and (8) into Eq. (2) gives the basic differential equation

$$\frac{dr}{dt} = a_r + m_r \left( \frac{(p_r - at - p_R)r_c}{2(R_0 - r(t))} - \frac{p_r - at + 3p_R}{4} \right). \quad (9)$$

The initial condition of Eq. (9) to be satisfied at time  $t_0 = 0$  is  $r(0) = r_0$ .

Solution of Eq. (9) provides the corresponding values of inner radius  $r$  and time  $t$ . For every values of  $r$  and  $t$ , the shell thickness can be found from (8) and stresses  $\sigma_r$  and  $\sigma_R$  can be calculated by Eqs. (6) and (7).

### 3.3 External Corrosion

In the case of external mechanochemical corrosion the outer radius decreases with time:  $R = R(t)$ , while the inner radius  $r$  remains constant:  $r = r_0 = \text{const}$ . Thus the shell thickness  $h$  at any time is

$$h = h(t) = R(t) - r_0. \quad (10)$$

Substituting Eqs. (7) and (10) into Eq. (3) yields

$$\frac{dR}{dt} = -a_R - m_R \left( \frac{(p_r - at - p_R)r_c}{2(R(t) - r_0)} - \frac{3p_r - 3at + p_R}{4} \right). \quad (11)$$

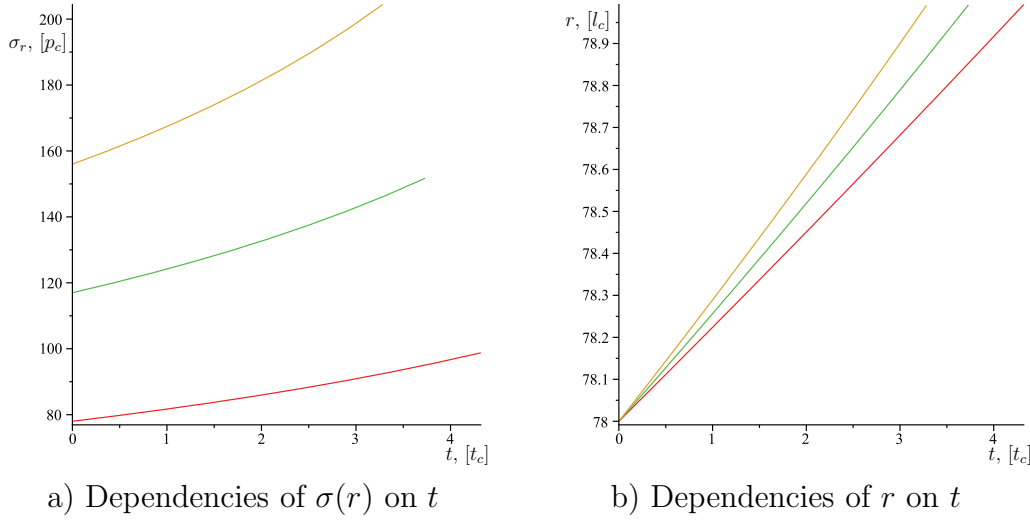
The initial condition to be satisfied at time  $t_0 = 0$  is  $R(0) = R_0$ .

Solution of Eq. (11) gives the corresponding values of outer radius  $R$  and time  $t$ . Shell thickness for a given  $R$  and  $t$  can be found from (10). Since  $R$ ,  $t$ , and  $h$  are found, stresses  $\sigma_r$  and  $\sigma_R$  can be calculated by Eqs. (6) and (7).

### 3.4 Lifetime Estimation

Let us define the lifetime of the shell as a minimum time  $t^*$  at which the equivalent stress in the shell reaches a given limit  $\sigma^*$ . The value  $\sigma^*$  supposed to be a strength limit or any other critical stress (taking into account safety factors) depending on the operating conditions [15, 16]. Note that this model is not applicable to local corrosion which can be caused by stress concentration in the vicinity of surface or near-surface defects [17, 18, 19, 21, 22, 20].

Note that, here we consider only the situations where  $p_{in} > p_{out}$ , i.e.  $p_r - at > p_R$ . For such cases the equivalent stress on inner shell's surface is greater than the outer stress:  $\sigma(r) > \sigma(R)$ . Thus, the shell's lifetime  $t^*$  is a solution of equation  $\sigma(r) = \sigma^*$ .



**Figure 1:** Internal corrosion. Effect of the initial internal pressure:  $p_r = 8[p_c]$  (red curves),  $p_r = 12[p_c]$  (green curves), and  $p_r = 16[p_c]$  (yellow curves)

#### 4 CALCULATION RESULTS

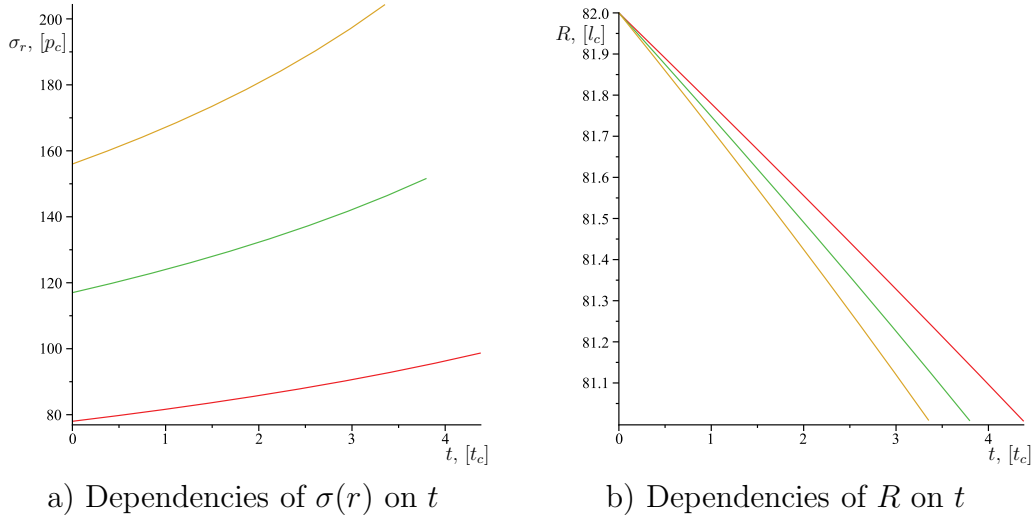
An example of the effect of the initial internal pressure on the maximum stress and the radii of the corroded shell is shown in figures 1 and 2. A spherical shell is considered with the initial radii  $R_0 = 82[l_c]$  and  $r_0 = 78[l_c]$  subjected to internal pressure  $p_{in} = p_r - at$  and external pressure  $p_R = 0$ . Different values of initial internal pressure are used:  $p_r = 8[p_c]$  (red curves),  $p_r = 12[p_c]$  (green curves), and  $p_r = 16[p_c]$  (yellow curves),  $p_c$  is a certain unit of stress; pressure decline parameter  $a = 0.1[p_c/t_c]$ . Here and below,  $p_c$ ,  $t_c$  and  $l_c$  is a certain unit of stress, time and length.

In figure 1 the dependencies of  $\sigma(r)$  on  $t$  (a) and  $r$  on  $t$  (b) are demonstrated for internal corrosion of the sphere. Corrosion rate parameters are  $a_r = 0.16[l_c/t_c]$  and  $m_r = 0.0008[l_c/(t_cp_c)]$ .

Figure 2 shows the dependencies of  $\sigma(r)$  on  $t$  (a) and  $R$  on  $t$  (b) in case of external corrosion. Corrosion rate parameters are  $a_R = 0.16[l_c/t_c]$  and  $m_R = 0.0008[l_c/(t_cp_c)]$ .

From figures 1 (a) and 2 (a) it is seen that for a fixed value  $a$ , the greater the initial value of  $p_{in}$  is, the greater  $\sigma(r)$  is, and the earlier a certain stress value is reached. Figure 1 (b) demonstrates that the inner radius increases with time due to corrosion, and the greater the initial value of  $p_{in}$  is, the faster  $r$  increases. Figure 2 (b) illustrates that the outer radius of the shell decreases with time, and the greater  $p_r$  is, the faster  $R$  decreases. Thus, in both cases of internal and external corrosion, an increase in the initial value of  $p_r$  accelerates the process of dissolution.

Figures 3 and 4 illustrate the effect of parameter  $a$  on the maximum stress and the radii of the sphere exposed to internal (fig. 3) and external (fig. 4) corrosion.



**Figure 2:** External corrosion. Effect of the initial internal pressure:  $p_r = 8[p_c]$  (red curves),  $p_r = 12[p_c]$  (green curves), and  $p_r = 16[p_c]$  (yellow curves)

In figure 3 the dependencies of  $\sigma(r)$  on  $t$  (a) and  $r$  on  $t$  (b) are shown. Figure 4 shows the dependencies of  $\sigma(r)$  on  $t$  (a) and  $R$  on  $t$  (b). Curves in figures 3 and 4 are built for pressures  $p_r = 30[p_c]$ ,  $p_R = 0$  and different values of parameter  $a$ :  $a = 4.5$ ,  $a = 3$ ,  $a = 1.5$ , and  $a = 0$ . All other parameters are the same as in figures 1 and 2.

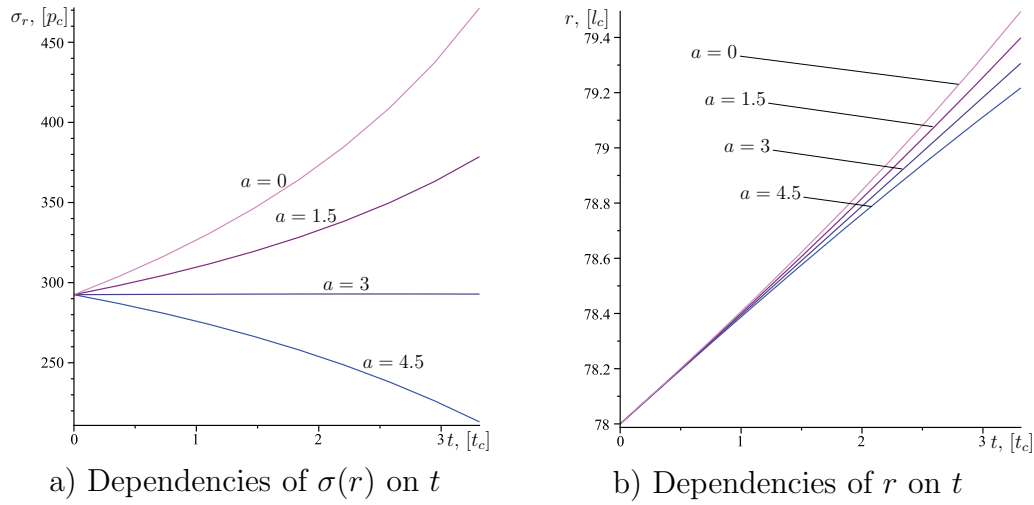
Figures 3 (a) and 4 (a) show that the stress in the shell grows slower for greater  $a$ , and can even diminishes at sufficiently high values of  $a$ . From figures 3 (b) and 3 (b) it is seen that the sphere's thickness reduces faster as  $a$  decreases. Thus, the larger  $a$  is, the greater the lifetime is.

## 5 CONCLUSIONS

- A linearly elastic thin-walled closed spherical shell exposing to mechanochemical corrosion under constant external pressure and internal pressure decreasing with time is considered. Corrosion rates are assumed to be linearly depend on stress.
- It is confirmed that the greater the initial value of internal pressure  $p_r$  is, the faster the corrosion process.
- The stress in the shell grows slower at a higher rate of the decrease of internal pressure. At sufficiently high rate of the decrease of internal pressure the stress in the shell may decline with time.

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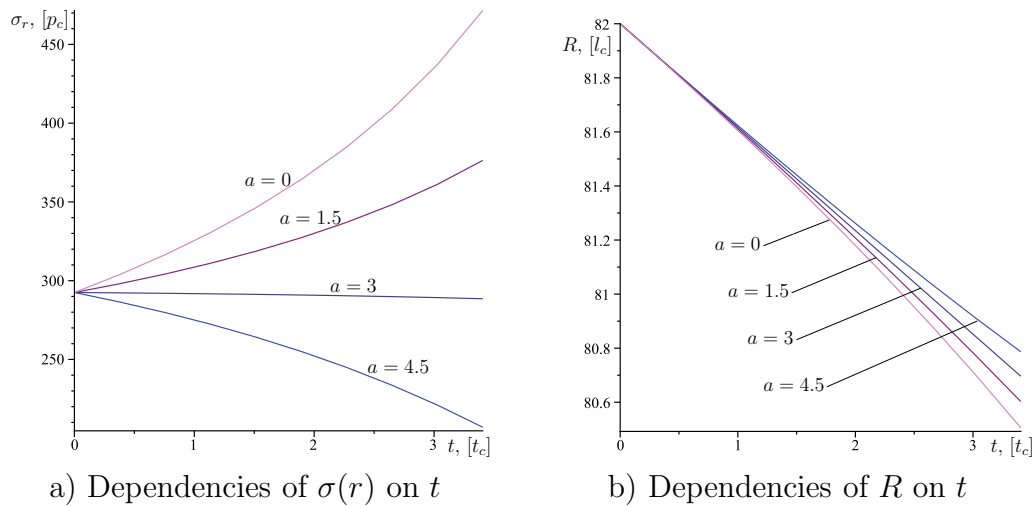
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**Figure 3:** Internal corrosion. Effect of various values of  $a$

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**Figure 4:** External corrosion. Effect of various values of  $a$

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